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Reentrance phenomenon of superfluid pairing in hot rotating nuclei

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Abstract. Reentrance phenomenon of superfluid pairing in highly rotating excited (hot) nuclei is studied within the Bardeen-Cooper-Schrieffer (BCS)-based approach (the FTBCS1), taking into account the effect of quasiparticle number fluctuations (QNF) on the pairing field at finite temperature and angular momentum within the pairing model plus noncollective rotation along the symmetry axis. The numerical calculations are carried out for the pairing gaps and level densities in ^{104}Pd, of which an anomalous enhancement of level density at low excitation energy E^* and high angular momentum J has been experimentally observed. The results obtained show that the QNF within the FTBCS1 theory smooth out the superfluid-normal phase transition and lead to the appearance of pairing reentrance phenomenon in hot rotating nuclei. This feature can be clearly seen in the behavior of pairing gap obtained at low E^* and high J. The good agreement between the level densities obtained within the FTBCS1 and those extracted from the experiment indicates that the observed enhancement in the level densities of ^{104}Pd nucleus is a manifestation of the pairing reentrance phenomenon.

1. Introduction

When a nucleus rotates (total angular momentum J and/or rotational frequency ω are not zero), the nucleon (proton and neutron) pairs located around the Fermi surface will scatter to the empty levels nearby and lead to the decreasing of pairing correlation. When the J or ω is sufficiently high, i.e., equal to the critical value J_c or ω_c, the scattered nucleons block completely the single-particle levels around the Fermi surface. Consequently, pairing correlation disappears. This phenomenon is called the Mottelson-Valatin effect [1], which is qualitatively similar as the collapsing of pairing correlation at the critical temperature T_c, where the superfluid-normal (SN) phase transition occurs. However, when J is slightly higher than J_c (or $\omega \geq \omega_c$), the increase of temperature T will relax the particles scattered around the Fermi surface and causes some levels become partially unoccupied, and therefore available for scattered pairs. As a result, when T increases up to a critical value T_1, the pairing correlation reappears. As T goes higher, e.g., at $T_2 > T_1$, the newly created pairs will again break and therefore the pairing correlation is eventually broken down. This phenomenon, which was first predicted by Kammuri [2] and Moretto [3], is called thermally assisted pairing correlation or anomalous pairing, and later as pairing reentrance by Balian, Flocard, and Veneroni [4]. It is similar to the Meissner effect in the superconducting metal in the presence of an external magnetic field, where the magnetic field plays the role as that of nuclear rotation [5].

It has been shown in the 1960s that the SN phase transition is an artifact of the BCS method because it neglects the thermal fluctuations in finite systems. The latter have been shown to be large so that they smooth out the SN phase transition in finite nuclei [6]. As the result, the pairing gap does not collapse at T_c , but monotonically decreases with increasing T and remains finite at $T \geq T_c$. The calculations within an exactly solvable pairing Hamiltonian for clusters and nuclei [7] and the cranked deformed shell model [8] for a single j-shell have predicted the pairing reentrance effect but in the way that pairing gap, which is zero at $J > J_c$ and $T = 0$, reappears at a certain T and does not vanish as predicted by the conventional FTBCS theory. The recently developed FTBCS1 theory that includes the effect due to quasiparticle-number fluctuations in the pairing field and angular momentum z projection at $T \neq 0$ has also predicted a similar behavior of pairing reentrance effect in some realistic nuclei [9, 10, 11]. In addition, the shell-model Monte Carlo calculations for heated rotating ^{72}Ge nucleus have suggested for the first time that the pairing reentrance effect can be seen not only in the behavior of pairing gap as functions of T and J (or ω) but also in the nuclear level density, an experimentally observed quantity, in a form of a local maximum at low T (or excitation energy E^*) and high J (or ω) [12]. Recently, by fitting the proton spectra of ^{12}C + ^{93}Nb $\rightarrow$ ^{105}Ag* $\rightarrow$ ^{104}Pd* + p reaction at the incident energy of 40 - 45 MeV, an enhancement of level density of ^{104}Pd at low E^* and high J (approximately greater than 17 $\hbar$) has been reported [13]. This enhancement is qualitatively similar to that predicted by the shell-model Monte Carlo simulation for ^{72}Ge and therefore might come from the pairing reentrance. The goal of the present work is to answer the question whether the enhancement observed in the extracted level density of ^{104}Pd is the first evidence of pairing reentrance phenomenon in atomic nuclei. For that purpose, we employ the FTBCS1 theory at finite temperature and angular momentum, in which the thermal fluctuations are included.

2. FTBCS1 theory at finite temperature and finite angular momentum

The FTBCS1 equations at finite temperature and angular momentum are derived based on the variational method to minimize the expectation value of the pairing Hamiltonian

$$H = \sum_k \epsilon_k (a^\dagger_{+k} a_{+k} + a^\dagger_{-k} a_{-k}) - G \sum_{kk'} a^\dagger_k a^\dagger_{-k} a_{-k'} a_{k'} - \lambda \hat{N} - \omega \hat{M} \ , \tag{1}$$

in the grand-canonical ensemble [3, 10]. Here, the Hamiltonian (1) describes a system rotating about the symmetry axis, which is chosen to coincide with its z component. The particle-number operator $\hat{N}$ and the z projection $\hat{M}$ of the total angular momentum $\hat{J}$ (which coincides with $\hat{M}$ for spherical nuclei) are defined as

$$\hat{N} = \sum_k (a^\dagger_{+k} a_{+k} + a^\dagger_{-k} a_{-k}) \ , \qquad \hat{M} = \sum_k m_k (a^\dagger_{+k} a_{+k} - a^\dagger_{-k} a_{-k}) \ , \tag{2}$$

where $a^\dagger_{\pm k}(a_{\pm k})$ are the creation (annihilation) operators of a particle in the k-th deformed state, whereas ϵ_k, λ, and ω are respectively the single-particle energies, chemical potential, and rotational frequency. The FTBCS1 equation for the pairing gap has the final form as [10, 11]

$$\Delta_k = \Delta + \delta\Delta_k \ , \tag{3}$$

where

$$\Delta = G\sum_{k'} u_{k'}v_{k'}(1-n_{k'}^{+}-n_{k'}^{-}) \ , \quad \delta\Delta_k = G\frac{\delta\mathcal{N}_k^2}{1-n_k^{+}-n_k^{-}}u_kv_k \ , \tag{4}$$

$$\begin{aligned} u_k^2 &= \frac{1}{2}\left(1+\frac{\epsilon_k-Gv_k^2-\lambda}{E_k}\right) \ , \quad v_k^2=\frac{1}{2}\left(1-\frac{\epsilon_k-Gv_k^2-\lambda}{E_k}\right) \ , \\ E_k &= \sqrt{(\epsilon_k-Gv_k^2-\lambda)^2+\Delta_k^2} \ , \quad n_k^{\pm}=\frac{1}{1+e^{\beta(E_k\mp\omega m_k)}} \ , \quad \beta=1/T \ , \end{aligned} \tag{5}$$

with $\delta\mathcal{N}_k^2$ being the quasi-particle-number fluctuations (QNF)

$$\delta\mathcal{N}_k^2=(\delta\mathcal{N}_k^{+})^2+(\delta\mathcal{N}_k^{-})^2=n_k^{+}(1-n_k^{+})+n_k^{-}(1-n_k^{-}) \ . \tag{6}$$

The equations for the particle number and total angular momentum are given as

$$N=2\sum_k\left[v_k^2(1-n_k^{+}-n_k^{-})+\frac{1}{2}(n_k^{\dagger}+n_k^{-})\right], \quad M=\sum_k m_k(n_k^{+}-n_k^{-}) \ . \tag{7}$$

In Eq. (4), if $\delta\mathcal{N}_k^2$ is set to be zero, i.e., no QNF, one recovers the conventional FTBCS equations at finite T and M. The total level density of a system with N neutrons and Z protons and energy $\mathcal{E}$ is obtained by using the inverse Laplace transformation of the grand partition function [3]. It reads

$$\rho(\mathcal{E},M)=\frac{e^{(S_N+S_Z)}}{(2\pi)^2\sqrt{D}} \ , \tag{8}$$

$$D=\begin{vmatrix} \frac{\partial^2\Omega}{\partial\alpha_N^2} & \frac{\partial^2\Omega}{\partial\alpha_N\partial\alpha_Z} & \frac{\partial^2\Omega}{\partial\alpha_N\partial\mu} & \frac{\partial^2\Omega}{\partial\alpha_N\partial\beta} \\ \frac{\partial^2\Omega}{\partial\alpha_Z\partial\alpha_N} & \frac{\partial^2\Omega}{\partial\alpha_Z^2} & \frac{\partial^2\Omega}{\partial\alpha_Z\partial\mu} & \frac{\partial^2\Omega}{\partial\alpha_Z\partial\beta} \\ \frac{\partial^2\Omega}{\partial\mu\partial\alpha_N} & \frac{\partial^2\Omega}{\partial\mu\partial\alpha_Z} & \frac{\partial^2\Omega}{\partial\mu^2} & \frac{\partial^2\Omega}{\partial\mu\partial\beta} \\ \frac{\partial^2\Omega}{\partial\beta\partial\alpha_N} & \frac{\partial^2\Omega}{\partial\beta\partial\alpha_Z} & \frac{\partial^2\Omega}{\partial\beta\partial\mu} & \frac{\partial^2\Omega}{\partial\beta^2} \end{vmatrix} \ . \tag{9}$$

where the grand-partition function Ω, total energy $\mathcal{E}$, and entropy S are given as

$$\Omega=\Omega_N+\Omega_Z=S_N+S_Z+\alpha_NN+\alpha_ZZ+\mu M-\beta\mathcal{E} \ , \ \ \mathcal{E}=<H>=\frac{\partial\Omega}{\partial\beta} \ , \tag{10}$$

$$S=-\sum_k[n_k^{+}\ln n_k^{+}+(1-n_k^{+})\ln(1-n_k^{+})+n_k^{-}\ln n_k^{-}+(1-n_k^{-})\ln(1-n_k^{-})] \ , \tag{11}$$

where $\alpha=\beta\lambda$ and $\mu=\beta\omega$. The total level density $\rho(\mathcal{E})$ is indeed calculated as the sum of J-dependent level densities, namely $\rho(\mathcal{E})=\sum_J(2J+1)\rho(\mathcal{E},J)$ [14], where $\rho(\mathcal{E},J)$ is obtained by differentiating $\rho(\mathcal{E},M)$ [15]

$$\rho(\mathcal{E},J)=\rho(\mathcal{E},M=J)-\rho(\mathcal{E},M=J+1) \ . \tag{12}$$

3. Results

The numerical calculations are carried out for ^{104}Pd nucleus, for which the single-particle spectra are taken from the axially deformed Woods-Saxon potentials with the Blomqvist-Wahlborn parametrization at a fixed value of quadrupole deformation parameter $\beta_2=0.276$ [16]. The pairing constant G is adjusted so that the pairing gaps obtained within the FTBCS (FTBCS1) at $T=0$ for proton and neutron fit the empirical odd-even mass differences, namely

$\Delta_N = 1.26$ MeV and $\Delta_Z = 1.5$ MeV [17]. Figures 1 (a)-(b) show that the level weighted pairing gaps $\bar{\Delta} \equiv \sum_k \Delta_k/\Omega$ (with Ω being the sum of all single-particle levels) obtained within the conventional FTBCS (thin lines) at all J decrease with increasing E^* and collapses at some critical values E_c^*. Here E^* is calculated as $E^* = \mathcal{E}(T, M) - \mathcal{E}(0, M)$. There is no significant enhancement of the level densities obtained within the FTBCS [See e.g., dotted lines in Figs. 1 (c)-(f)] as those in the experimental data at $J \geq 20\hbar$. As a result, the FTBCS does not show the pairing reentrance effect at all J.

For the FTBCS1, due to the inclusion of the QNF in the gap equation (4), the FTBCS1 gaps decrease monotonically with increasing E^* and do not collapse at $E^* = E_c^*$ as in the case of the FTBCS. Instead the pairing gaps in the FTBCS1 remain finite even at $E^* > 15$ MeV. Within the FTBCS1, the pairing reentrance is seen very clearly at $J = 20\hbar$ for neutrons and at $J = 30\hbar$ for protons [See e.g., dashed lines in Figs. 1 (c)-(f)]. Consequently, there appear local enhancements in the FTBCS1 level densities at around $2 < E^* < 5$ MeV at these two values of J. The FTBCS1 level densities agree fairly well with the experimental data at all J values considered in present work.

It is worth noticing that the FTBCS1 employed in this work includes only the monopole pairing, whereas the higher multipolarities, which are responsible for the collective motion in finite nuclei and might lead to the increase of the level densities, are neglected in the residual interaction. We therefore do not expect an excellent quantitative agreement between the theoretical predictions and the experimental data. However, the qualitative physics in the present work is not affected by this neglect, that is, the enhancement observed in the experimentally extracted level density at low E^* and high J in ^{104}Pd nucleus is indeed an evidence of pairing reentrance.

4. Conclusions

The present work employs the FTBCS1 theory at finite temperature and angular momentum to study the pairing phenomenon and level density in ^{104}Pd, of which an enhancement of level density at low excitation energy and high angular momentum has been experimentally observed. The quantitative agreement between experiment and theory suggests that this enhancement is indeed the first experimental evidence of the reentrance of superfluid pairing in a finite nucleus. The numerical calculations were carried out using the Integrated Cluster of Clusters (RICC) system at RIKEN. This work is supported by the National Foundation for Science and Technology Development (NAFOSTED) of Vietnam through Grant No.103.04-2013.08.

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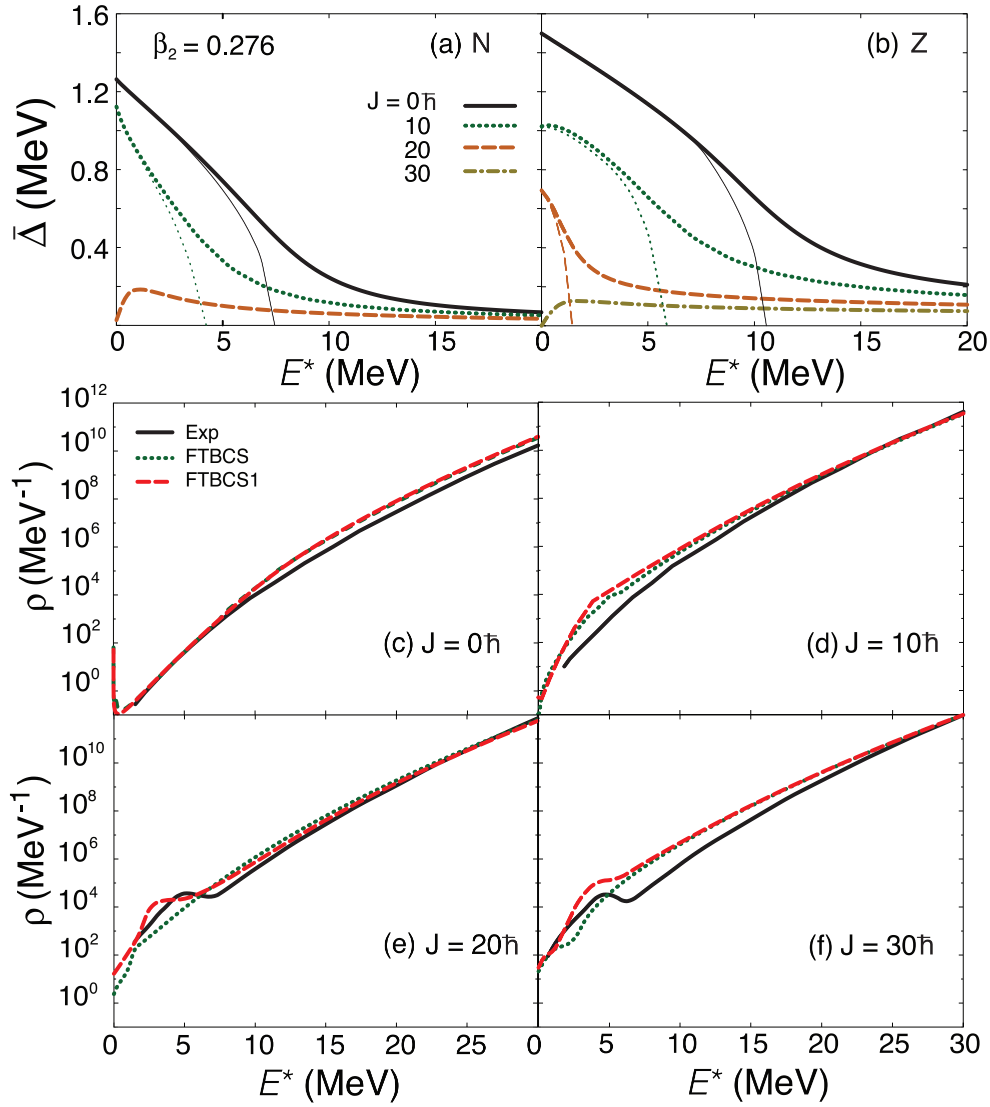

Figure 1. . Level-weighted pairing gaps $\bar{\Delta}$ for neutron (N) (a) and protons (Z) (b) and total level densities (c) - (f) as function of excitation energy E^* obtained within the FTBCS and FTBCS1 at the quadrupole deformation parameter $\beta_2 = 0.276$. The thin and thick lines in (a) and (b) denote the FTBCS and FTBCS1 results, respectively, whereas the dotted and dashed lines in (c) - (f) respectively stand for the FTBCS and FTBCS1 total level densities. The solid lines in (c) - (f) are the experimentally extracted data.

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